Reply by the Authors to Yurij P. Ulybyshev

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SEVERAL points are raised by Ulybyshev in the preceding Technical Comment. He correctly points out two typographical errors in the printed version of the article. These were introduced in the typesetting process and evaded proofreading.

The assertion is also made that Marinescu[1] was the first to present an analytical solution for the optimal thrust acceleration in the Hill–Clohessy–Wiltshire (often called CW) gravitational model in 1976. Whereas it is true that in Ref. 1 three earlier related conference papers in French are cited in the period 1965–68, Gobetz[2,3] presented the solution for the optimal thrust for this problem in 1964–65. Reference 3 is cited by Marinescu but utilizes orbital elements, whereas Ref. 2 utilizes the CW equations of motion. Solution using optimal control theory is straightforward because it is a linear-quadratic problem.

Ulybyshev points out in his Comment and in a previous comment[4] on Lembeck and Prussing[5] that the vector constants in the various forms of the solutions are related by linear transformations, which is of course true. Perhaps the simplest way of obtaining the optimal trajectory is to combine the equation of motion and the necessary conditions for optimality as in Ref. 6 to obtain a single, fourth-order differential equation for the position vector on an optimal trajectory.

For a linear system, such as Hill–Clohessy–Wiltshire, the equation of motion describing the position vector $\boldsymbol{r}$ is of the form

$$\ddot{\boldsymbol{r}} = \boldsymbol{A}\boldsymbol{r} + \boldsymbol{B}\dot{\boldsymbol{r}} + \boldsymbol{\Gamma} \tag{1}$$

where $\boldsymbol{\Gamma}(t)$ is the thrust acceleration and $\boldsymbol{A}$ and $\boldsymbol{B}$ are constant 3×3 matrices. Specializing the procedure in Ref. 6 to the case of a linear, time-invariant system (which is done in Ref. 7) provides the equation that the position vector satisfies when the optimal thrust acceleration is applied:

$$\boldsymbol{r}^{iv} - 2\boldsymbol{B}\boldsymbol{r}^{iii} + (\boldsymbol{B}^2 - 2\boldsymbol{A})\ddot{\boldsymbol{r}} + (\boldsymbol{A}\boldsymbol{B} + \boldsymbol{B}\boldsymbol{A})\dot{\boldsymbol{r}} + \boldsymbol{A}^2\boldsymbol{r} = \boldsymbol{0} \tag{2}$$

where $\boldsymbol{r}^{iii}$ and $\boldsymbol{r}^{iv}$ represent the third and fourth derivatives. Every solution to Eq. (2) is an optimal trajectory. The solution can be obtained analytically using Laplace transforms[8] and the desired boundary conditions satisfied.

References

[1]Marinescu, A., "Optimal Low-Thrust Orbital Rendezvous," *Journal of Spacecraft and Rockets*, Vol. 13, No. 7, 1976, pp. 385–392.

[2]Gobetz, F. W., "Optimal Variable-Thrust Transfer of a Power-Limited Rocket between Neighboring Circular Orbits," *AIAA Journal*, Vol. 2, No. 2, 1964, pp. 339–343.

[3]Gobetz, F. W., "A Linear Theory of Optimum Low-Thrust Rendezvous Trajectories," *Journal of the Astronautical Sciences*, Vol. 12, Fall 1965, pp. 69–76.

[4]Ulybyshev, Y. P., "Comment on Optimal Impulsive Intercept with Low-Thrust Rendezvous Return," *Journal of Guidance, Control, and Dynamics*.

[5]Lembeck, C. A., and Prussing, J. E., "Optimal Impulsive Intercept with Low-Thrust Rendezvous Return," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 3, 1993, pp. 426–433.

[6]Prussing, J. E., "Equation for Optimal Power-Limited Spacecraft Trajectories," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 2, 1993, pp. 391–393.

[7]Prussing, J. E., "Equation for Optimal Power-Limited Spacecraft Trajectories," *Advances in the Astronautical Sciences*, Vol. 82, Pt. 1, edited by R. G. Melton et al., American Astronomical Society, San Diego, CA, 1993, pp. 639–648.

[8]Murphy, S. P., "A Study of Optimal Power-Limited Spacecraft Trajectories," M.S. Thesis, Univ. of Illinois, Dept. of Aeronautical and Astronautical Engineering,Urbana–Champaign, IL, May 1992.

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